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# FUZZIFICATION OF LEGENDRE POLYNOMIALS

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**Abstract**
In this paper, we have considered basic Legendre polynomials obtained from Rodrigue's formula and observed the findings if we use fuzzy intervals citing the particular case of increasing order of the arguments and using triangular fuzzy number in the polynomials taken into consideration. Fuzzy membership functions are found out by adoption of different methods.

***Keywords:*** *fuzzy membership function(f.m.f.), triangular fuzzy number(tfn), interval of confidence, α-cuts..*

## I. INTRODUCTION

Legendre differential equations are included into the category of linear differential equations of second order with variable coefficients. In solving these equations, explicit solutions cannot be found. That is, solutions is in terms of elementary functions cannot be found. In many cases it is easier to find a numerical or series solution. This particular Differential Equation has got importance in applied mathematics, particularly in boundary value problems involving spherical configurations. Though n is a real number, only integral value of n is required in most physical applications. It is to be referred that the concept of Fuzzy differential equation was first introduced by Chang and Zadeh [1]. Dubois and Prade [2] has given the extension principle.

## II. BASIC CONCEPTS AND DEFINITIONS

A triangular Fuzzy number μ is defined by three real numbers with base as the interval [a,c] and b as the vertex of the triangle. The membership functions are defined as follows:

$$\mu(x)(=)\begin{cases}\dfrac{x-a}{b-a}; & where\ a \le x \le b\\ \dfrac{x-c}{b-c}; & where\ b \le x \le c\\ 0\ ; & otherwise\end{cases}$$

Where α-cuts are given by

$$\Delta_L(\alpha)(=)a+\alpha(b-a)$$

*and*

$$\Delta_R(\alpha)(=)c+\alpha(b-c)$$

## III. LEGENDRE POLYNOMIALS IN TERMS OF RODRIGUE'S FORMULA

Also as per Rodrigue formula , Legendre polynomial is

$$P_n(x) = \frac{1}{2^n.n!}\frac{d^n}{dx^n}(x^2-1)^n \qquad \text{-------------------------------------------(1)}$$

$$Putting \quad n=1, \qquad P_1(x)=\frac{1}{2^1.1!}\frac{d}{dx}(x^2-1)=\frac{1}{2}.2x=x$$

$$n=2, \qquad P_2(x)=\frac{1}{2^2.2!}\frac{d^2}{dx^2}(x^2-1)^2=\frac{1}{8}\frac{d^2}{dx^2}(x^4+1-2x^2)$$

$$=\frac{1}{8}(12x^2-4)=\frac{1}{2}(3x^2-1)$$

$$n=3, \qquad P_3(x)=\frac{1}{2^3.3!}\frac{d^3}{dx^3}(x^2-1)^3=\frac{1}{2}(5x^3-3x)$$

## IV. FUZZIFICATION OF LEGENDRE POLYNOMIAL $P_2(X)$

Let us Fuzzify the Legendre polynomial $P_2(x)$ where $P_2(x)=\frac{1}{2}(3x^2-1)$ --------------------------------(2)

Let $x(=)[x_1,x_2,x_3]$ such that membership function $\mu(x)(=)\left\{\begin{matrix} \frac{x-x_1}{x_2-x_1}; \; x_1\le x\le x_2 \\ \frac{-x+x_3}{x_3-x_2}; \; x_2\le x\le x_3 \\ 0 \qquad ;otherwise \end{matrix}\right\}$

Hence α-cut for x is $[x]^{(\alpha)}(=)[x_1+(x_2-x_1)\alpha,\; x_3-(x_3-x_2)\alpha]$, Now α-cut for $P_2(x)$ is

$$[P_2(x)]^{(\alpha)}(=)\frac{1}{2}\{3x(.)x-1\}(=)\frac{1}{2}\begin{Bmatrix} 3[x_1+(x_2-x_1)\alpha,\; x_3-(x_3-x_2)\alpha](.) \\ [x_1+(x_2-x_1)\alpha,\; x_3-(x_3-x_2)\alpha]-1 \end{Bmatrix}$$

$$(=)\frac{1}{2}\{3[\{x_1+(x_2-x_1)\alpha\}^2,\{x_3-(x_3-x_2)\alpha\}^2]-1\}$$

$$(=)\frac{1}{2}\left\{3\begin{bmatrix} x_1^{\,2}+2x_1(x_2-x_1)\alpha+(x_2-x_1)^2\alpha^2-1/3, \\ x_3^{\,2}-2x_3(x_3-x_2)\alpha+(x_3-x_2)^2\alpha^2-1/3 \end{bmatrix}\right\}$$

$$P_2(x)(=)\frac{1}{2}[3[x_1,x_2,x_3](.)[x_1,x_2,x_3]-1] \text{ assuming } 0\le x_1\le x_2\le x_3$$

Also $$(=)\frac{1}{2}[3[x_1^{\,2},x_2^{\,2},x_3^{\,2}]-1]$$

$$(=)\left[\frac{1}{2}(3x_1^{\,2}-1),\;\frac{1}{2}(3x_2^{\,2}-1),\;\frac{1}{2}(3x_3^{\,2}-1)\right]$$

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Where

$$\frac{3}{2}(x_2-x_1)^2\alpha^2+3x_1(x_2-x_1)\alpha+\frac{3}{2}(x_1^{\ 2}-\tfrac{1}{3})(=)X_1$$

and

$$\frac{3}{2}(x_3-x_2)^2\alpha^2-3x_3(x_3-x_2)\alpha+\frac{3}{2}(x_3^{\ 2}-\tfrac{1}{3})(=)X_2$$

Now we are to retain two roots $\alpha\in[0,1]$ such that

$$\therefore \quad \alpha(=)\frac{-3x_1(x_2-x_1)\pm\sqrt{9x_1^{\ 2}(x_2-x_1)^2-4.\frac{3}{2}(x_2-x_1)^2.\frac{3}{2}(x_1^{\ 2}-\frac{1}{3}-\frac{2}{3}X_1)}}{3(x_2-x_1)^2}$$

$$\text{and} \quad \alpha(=)\frac{3x_3(x_3-x_2)\pm\sqrt{9x_3^{\ 2}(x_3-x_2)^2-4.\frac{3}{2}(x_3-x_2)^2.\frac{3}{2}(x_3^{\ 2}-\frac{1}{3}-\frac{2}{3}X_2)}}{3(x_3-x_2)^2}$$

Hence f.m.f for $P_2(x)$ is

$$\mu_{P_2(x)}(X)=\begin{cases}\dfrac{-x_1(x_2-x_1)+\sqrt{x_1^{\ 2}(x_2-x_1)^2-(x_2-x_1)^2(x_1^{\ 2}-\frac{1}{3}-\frac{2}{3}X)}}{(x_2-x_1)^2}, \\ \quad where\ \frac{1}{2}(3x_1^{\ 2}-1)\le X\le\frac{1}{2}(3x_2^{\ 2}-1) \\ \dfrac{x_3(x_3-x_2)-\sqrt{x_3^{\ 2}(x_3-x_2)^2-(x_3-x_2)^2(x_3^{\ 2}-\frac{1}{3}-\frac{2}{3}X)}}{(x_3-x_2)^2}, \\ \quad where\ \frac{1}{2}(3x_2^{\ 2}-1)\le X\le\frac{1}{2}(3x_3^{\ 2}-1) \\ 0, \quad otherwise\end{cases}$$

## V. FUZZIFICATION OF LEGENDRE POLYNOMIAL $P_3$(X)

Next, let us Fuzzify the Legendre polynomial $P_3(x)$ where

$$P_3(x)=\frac{1}{2^3.3!}\frac{d^3}{dx^3}(x^2-1)^3=\frac{1}{2}(5x^3-3x) \qquad \text{------------------------------------------(3)}$$

Now α-cut for $P_3(x)$ is

$$[P_3(x)]^{(\alpha)}(=)\frac{1}{2}[5x(.)x(.)x-3x](=)\frac{1}{2}\begin{bmatrix}5[\{x_1+(x_2-x_1)\alpha\}^3,\{x_3-(x_3-x_2)\alpha\}^3]\\-3[\{x_1+(x_2-x_1)\alpha\},\{x_3-(x_3-x_2)\alpha\}]\end{bmatrix}$$

$$(=)\frac{1}{2}\begin{bmatrix}5\begin{Bmatrix}x_1^3+3x_1^2k_1\alpha+3x_1k_1^2\alpha^2+k_1^3\alpha^3, x_3^3-3x_3^2k_2\alpha\\+3x_3k_2^2\alpha^2-k_2^3\alpha^3\end{Bmatrix}\\+\{-3\{x_3-(x_3-x_2)\alpha\},-3\{x_1+(x_2-x_1)\alpha\}\}\end{bmatrix}$$

$$(=)\frac{1}{2}\begin{bmatrix}5x_1^3-3x_3+(15x_1^2k_1+3k_2)\alpha+15x_1k_1^2\alpha^2+5k_1^3\alpha^3,\\5x_3^3-3x_1-(15x_3^2k_2+3k_1)\alpha+15x_3k_2^2\alpha^2-5k_2^3\alpha^3\end{bmatrix}$$

where $x_{n+1}-x_n=k_n \quad n=1,2$

$$(=)\left[F\left(X^{/}\right),F\left(X^{//}\right)\right] \quad \text{(Say)}$$

Now putting the values $\alpha \,\varepsilon\, [0,1]$ and hence respective values of $F(X^{/})$ and $F(X^{//})$ are shown in Table 1.

***Table 1***

| $\alpha$ | $F(X^{/})$ | $F(X^{//})$ |
|---|---|---|
| 0 | $\frac{1}{2}\left(5x_1^3-3x_3\right)=l_1\ (say)$ | $\frac{1}{2}\left(5x_3^3-3x_1\right)=m_1\ (say)$ |
| .25 | $\frac{1}{2}\begin{Bmatrix}(5x_1^3-3x_3)+\frac{1}{4}(15x_1^2k_1+3k_2)\\+\frac{15}{16}x_1k_1^2+\frac{5}{64}k_1^3\end{Bmatrix}$ $=l_2\ (say)$ | $\frac{1}{2}\begin{Bmatrix}(5x_3^3-3x_1)-\frac{1}{4}(15x_3^2k_2+3k_1)\\+\frac{15}{16}x_3k_2^2-\frac{5}{64}k_2^3\end{Bmatrix}$ $=m_2\ (say)$ |
| .5 | $\frac{1}{2}\begin{Bmatrix}(5x_1^3-3x_3)+\frac{1}{2}(15x_1^2k_1+3k_2)\\+\frac{15}{4}x_1k_1^2+\frac{5}{8}k_1^3\end{Bmatrix}$ $=l_3\ (say)$ | $\frac{1}{2}\begin{Bmatrix}(5x_3^3-3x_1)-\frac{1}{2}(15x_3^2k_2+3k_1)\\+\frac{15}{4}x_3k_2^2-\frac{5}{8}k_2^3\end{Bmatrix}$ $=m_3\ (say)$ |
| .75 | $\frac{1}{2}\begin{Bmatrix}(5x_1^3-3x_3)+\frac{3}{4}(15x_1^2k_1+3k_2)\\+\frac{135}{16}x_1k_1^2+\frac{135}{64}k_1^3\end{Bmatrix}$ $=l_4\ (say)$ | $\frac{1}{2}\begin{Bmatrix}(5x_3^3-3x_1)-\frac{3}{4}(15x_3^2k_2+3k_1)\\+\frac{135}{16}x_3k_2^2-\frac{135}{64}k_2^3\end{Bmatrix}$ $=m_4\ (say)$ |
| 1 | $\frac{1}{2}\begin{Bmatrix}(5x_1^3-3x_3)+(15x_1^2k_1+3k_2)\\+15x_1k_1^2+5k_1^3\end{Bmatrix}$ $=l_5\ (say)$ | $\frac{1}{2}\begin{Bmatrix}(5x_3^3-3x_1)-(15x_3^2k_2+3k_1)\\+15x_3k_2^2-5k_2^3\end{Bmatrix}$ $=m_5\ (say)$ |

Using Lagrange's interpolation formula

$$F(X^{/})(=)\frac{(x-l_2)(x-l_3)(x-l_4)(x-l_5)}{(l_1-l_2)(l_1-l_3)(l_1-l_4)(l_1-l_5)}\alpha_1+\frac{(x-l_1)(x-l_3)(x-l_4)(x-l_5)}{(l_2-l_1)(l_2-l_3)(l_2-l_4)(l_2-l_5)}\alpha_2$$
$$+\frac{(x-l_1)(x-l_2)(x-l_4)(x-l_5)}{(l_3-l_1)(l_3-l_2)(l_3-l_4)(l_3-l_5)}\alpha_3+\frac{(x-l_1)(x-l_2)(x-l_3)(x-l_5)}{(l_4-l_1)(l_4-l_2)(l_4-l_3)(l_4-l_5)}\alpha_4$$
$$+\frac{(x-l_1)(x-l_2)(x-l_3)(x-l_4)}{(l_5-l_1)(l_5-l_2)(l_5-l_3)(l_5-l_4)}\alpha_5$$

And

$$F(X^{//})(=)\frac{(x-m_2)(x-m_3)(x-m_4)(x-m_5)}{(m_1-m_2)(m_1-m_3)(m_1-m_4)(m_1-m_5)}\alpha_1$$
$$+\frac{(x-m_1)(x-m_3)(x-m_4)(x-m_5)}{(m_2-m_1)(m_2-m_3)(m_2-m_4)(m_2-m_5)}\alpha_2$$
$$+\frac{(x-m_1)(x-m_2)(x-m_4)(x-m_5)}{(m_3-m_1)(m_3-m_2)(m_3-m_4)(m_3-m_5)}\alpha_3$$
$$+\frac{(x-m_1)(x-m_2)(x-m_3)(x-m_5)}{(m_4-m_1)(m_4-m_2)(m_4-m_3)(m_4-m_5)}\alpha_4$$
$$+\frac{(x-m_1)(x-m_2)(x-m_3)(x-m_4)}{(m_5-m_1)(m_5-m_2)(m_5-m_3)(m_5-m_4)}\alpha_5$$

Hence fuzzy membership function for $P_3(x)$ is

$$\mu_{P_3(x)}(x)(=)\begin{cases} F(X^{/})\ ;\ where\ l_1\le x\le l_5 \\ F(X^{//})\ ;\ where\ m_5\le x\le m_1 \\ 0\qquad ;\ Otherwise \end{cases}$$

## VI. CONCLUSION

Here we have discussed the fuzzy solution of Legendre polynomials P2(x) and P3(x). Fuzzy membership functions of these functions are obtained which will submit the fuzziness of respective functions in specified intervals.

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